LESSON 6.1a

Exponential Growth and Decay

Today you will:

- Learn what exponential functions are and how to graph them.
- Learn what exponential growth and decay functions are and how they are used in the real world.
- Practice using English to describe math processes and equations

Core Vocabulary:

- Exponential function, p. 296
- Exponential growth function, p. 296
- Growth factor , p. 296
- Exponential decay function , p. 296
- Decay factor , p. 296
- Asymptote , p. 296

Previous:

- Properties of exponents

Exponential Function

Variable is in the exponent!

$$y = ab^{x}$$

Leading coefficient

Base

Not a variable, just a number

Examples of exponential functions:

- $y = 2^x$ $a = 1,$ $b = 2$
- $y = 3(1.2)^x$ $a = 3,$ $b = 1.2$
- $y = 1.3(.72)^x$ $a = 1.3,$ $b = .72$

When $a \neq 1$ we put parenthesis around b

Exponential Growth Function

Exponential function where $a > 0$ and $b > 1$

Example: $y = 2(1.1)^x$

Exponential Decay Function

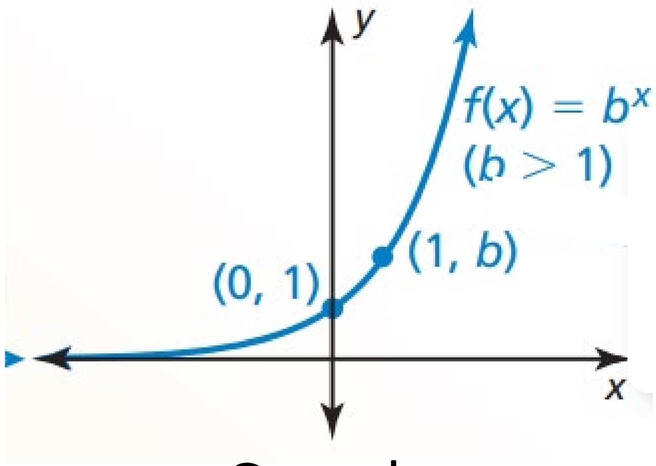
Exponential function where $a > 0$ and $0 < b < 1$,
i.e. b is a positive decimal/fraction less than 1

Example: $y = 4(0.32)^x$

What do graphs of exponential functions look like?

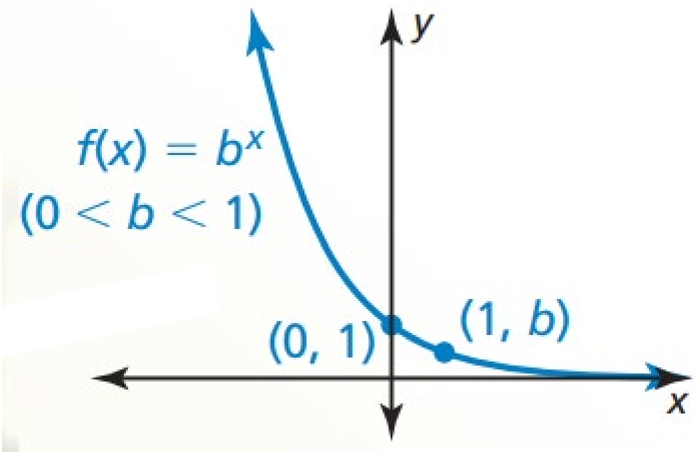
- Parent function for exponential functions is:

$$f(x) = b^x \quad (a = 1)$$



Growth

b is referred to as the ***growth factor***



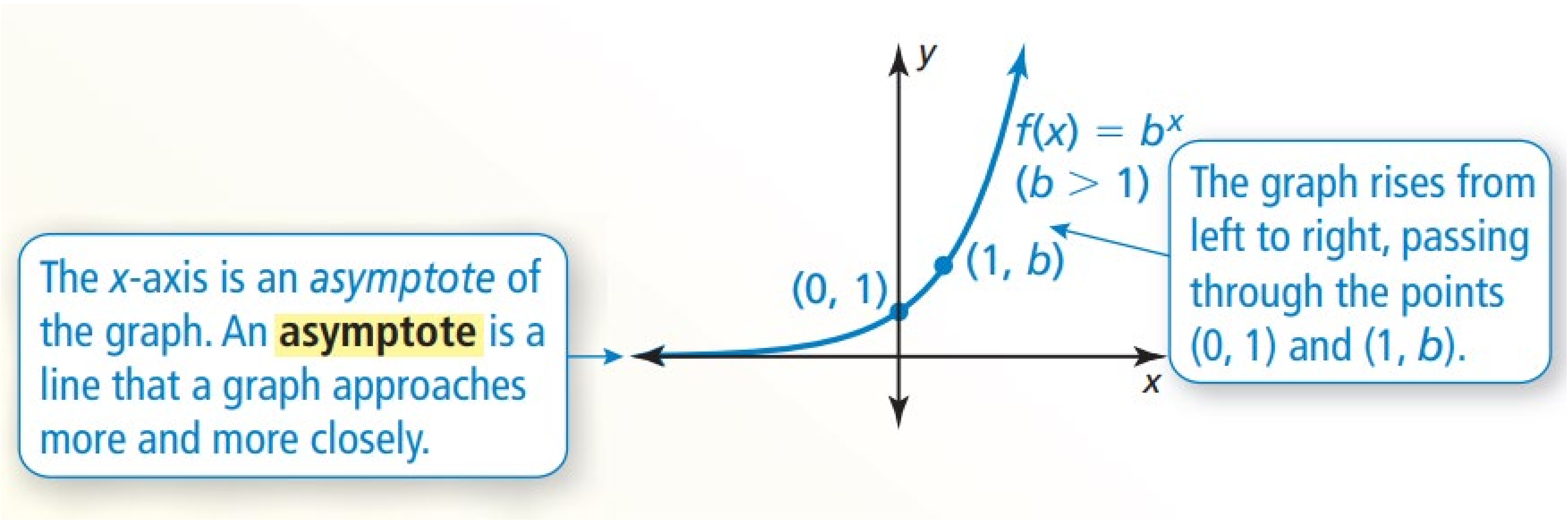
Decay

b is referred to as the ***decay factor***

Definition: Asymptote

A line that a graph approaches, gets closer and closer to but never touches.

For exponential functions, the x-axis is an ***asymptote***



Tell whether each function represents *exponential growth* or *exponential decay*. Then graph the function.

a. $y = 2^x$

b. $y = \left(\frac{1}{2}\right)^x$

SOLUTION

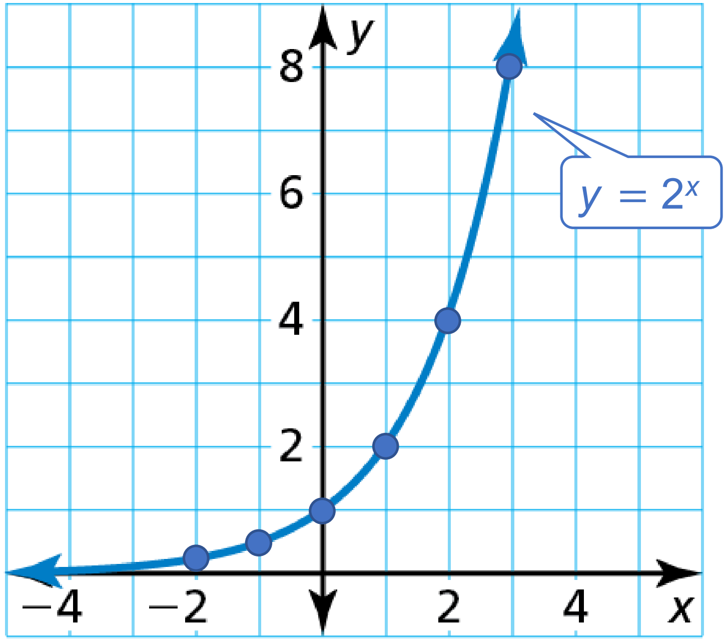
a. Step 1 Identify the value of the base. The base, 2, is greater than 1, so the function represents exponential growth.

Step 2 Make a table of values.

x	−2	−1	0	1	2	3
y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

Step 3 Plot the points from the table.

Step 4 Draw, from *left to right*, a smooth curve that begins just above the *x*-axis, passes through the plotted points, and moves up to the right.



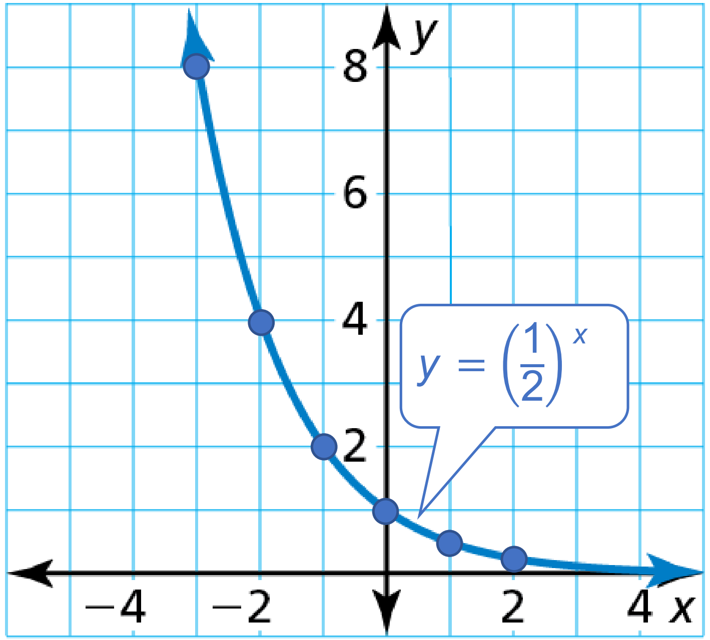
b. Step 1 Identify the value of the base. The base, $\frac{1}{2}$, is greater than 0 and less than 1, so the function represents exponential decay.

Step 2 Make a table of values.

x	−3	−2	−1	0	1	2
y	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$

Step 3 Plot the points from the table.

Step 4 Draw, from *right to left*, a smooth curve that begins just above the *x*-axis, passes through the plotted points, and moves up to the left.



Exponential Models

Exponential equations/functions that reflect (model) real-world situations.

A common example is a value you are tracking that increases or decreases by a fixed percentage over a regular period of time (a year).

Exponential Growth Model	**Exponential Decay Model**
$y = a(1 + r)^t$	$y = a(1 - r)^t$

…where a is the initial/starting value
r is the rate of change
t is the amount of time that has past

$(1 + r)$ is the growth factor
$(1 - r)$ is the decay factor

The value of a car y (in thousands of dollars) can be approximated by the model $y = 25(0.85)^t$, where t is the number of years since the car was new.

a. Tell whether the model represents exponential growth or exponential decay.

b. Identify the annual percent increase or decrease in the value of the car.

c. Estimate when the value of the car will be \$8000.

REASONING QUANTITATIVELY

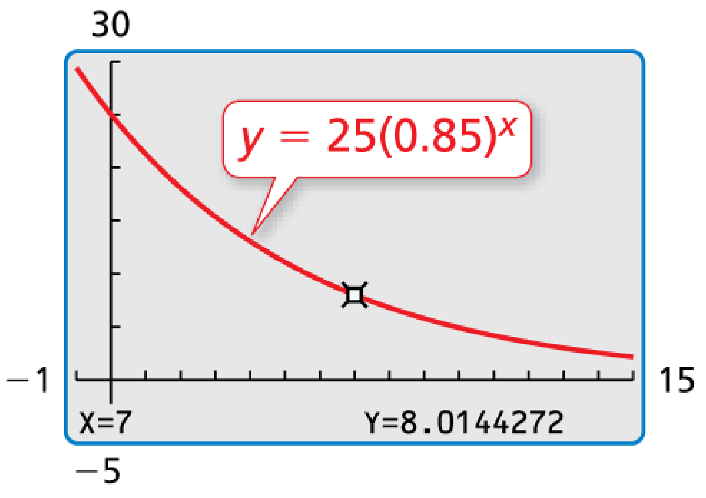
The percent decrease, 15%, tells you how much value the car *loses* each year. The decay factor, 0.85, tells you what fraction of the car's value *remains* each year.

SOLUTION

a. The base, 0.85, is greater than 0 and less than 1, so the model represents exponential decay.

b. Because t is given in years and the decay factor $0.85 = 1 - 0.15$, the annual percent decrease is 0.15, or 15%.

c. Use the *trace* feature of a graphing calculator to determine that $y \approx 8$ when $t = 7$. After 7 years, the value of the car will be about \$8000.



Homework

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